SYLLABUS

B.Com I Year

Subject – Business Mathematics

UNIT – I	Ratio – Gaining and Sacrificing Ratio, Proportion, Percentage, Commission, Discount and Brokerage.
UNIT – II	Simultaneous Equations – Meaning, Characteristic, Types and Calculations, Preparation of Invoice.
UNIT – III	Elementary Matrices – Definition and Calculations, Types of Matrices.
UNIT – IV	Logarithms and Antilogarithms – Principles and Calculations, Simple and Compound Interest.
UNIT – V	Average – Simple, Weighted and Statistical Averages Arithmetic mean, Harmonic mean, Geometric mean, Profit and Loss.

UNIT-I
Chapter -1 – RATIO

A ratio can exist only between two quantities of the same type. If x and y are any two numbers and y $\neq 0$ then the fraction $\frac{x}{y}$ is called the ratio of x and y is written as x:y.

Characteristics of Ratio –

The following characteristics are attributed to ratio relationship:

    i)   Ratio is a cross relation found between two or more quantities of same type.

    ii)   It must be expressed in the same units.

    iii)   By the fraction laws a ratio can be expressed as below:

$$\frac{y}{x} = x : y$$

$$\frac{10}{5} = 10 : 5 \text{ or } 2 : 1$$

    iv)   A ratio expresses the number of times that one quantity contains another.

    v)   Two or more ratios may be compared by reducing their equivalent fractions to a common denominator.

Different types of Ratio –

Ratio can be divided into following ways –

1) Unit Ratio – When homogeneous items are same on the basis of unit, it is called unit ratio.
   For example – Ram and Shyam are getting Rs. 5 each.
   $$\frac{x}{y} = \frac{5}{5} \text{ or } 5 : 5 \text{ or } 1 : 1$$

2) Duplicate Ratio – When the homogeneous items are shown in unit with square, it is called duplicate ratio.
   For Example, 2:3 square means $2^2 : 3^2$ or 4:9

3) Triplicate ratio – When homogenous item is multiplied by 3, it is known as triplicate ratio.
   For example, $2^3 : 3^3$ = x2x2x2:3x3x3   8:27

4) Sub triplicate ratio – When ratio is expressed in cube root it is known as sub triplicate ratio.
   For example, $\sqrt[3]{8} : \sqrt[2]{27}$ = 2:3

5) Ratio of greater in equality – In this type of ratio the first item of given ratio is greater than other items.
   For example,   8:3, 13:8.

6) Ratio of less in equality – When first item of given ratio is less than the other items of ratio, it is called ratio of less of equality.
   For example,   2:7, 5:12, 1:3

7) Equality ratio – In this type of ratio first item is equal to other item of ratio.
   For example,   5:5, 8:8, 12:12

Proportion

Relationship between the two ratio's is called proportion. Here, quantity ratio of first two items is equality to rest two terms.
        For example,   2:5::6:15
        Proportion is expressed by four parallel points (::).

In the simple proportion here its not necessary that two items of first ratio and the items of second ratio should be homogeneous. But the items of second set of ratio has the same relationship which is found between the items of first ratio. For example 2:5::6:15. Here 5 is 2.5 times of 2 in case of first ratio. In the same 15 is 2.5 times of 6 in the second set of ratio.

Characteristics of Proportion –
   i)   Proportion is given in four parts. So first number is known as first item, second number is second item, third number is third item and fourth number is known as fourth item.
   ii)   First and fourth items are known as extremes items and second and third items are known as mean items.
   iii) It is not necessary in proportion that all four items should be homogenous. But the ratios of first and second and third and fourth should be the same.

Types of Proportion –

1) **Continued proportion** –
   If ratio of items is going on continuously, e.g., ratio of first and two is equal to two and three and ratio of two and three is equal to three and fourth item and so on, thus, ratio is known as continued ratio.
   For example, $\frac{A}{B} = \frac{B}{C} = \frac{C}{D} = \frac{D}{E} = \frac{E}{F} \dots$
   Here A, B, C, D, E and F are in continued ratio.
2) **Direct Proportion** –
   In this type of ratio, two different items has the such relation that if the one is increased or decreased,   another will change accordingly in the same ratio.

Difference Between Ratio and Proportion –

S.No.	Ratio	Proportion
1	There are two terms in a ratio.	There are four terms in a proportion.
2	Comparison of two quantities of same type.	Comparison of two ratios.
3	Two quantities must be of same type.	All four quantities are not of same type but the first two are of one type and the last two may be of another type.
4	There is not a product rule	The product of extremes is equal to product of the means.

Chapter -2 - PERCENTAGE
Percent and Percentage
When we take of percentage, we usually refer to "for every one hundred."
Actually percentage can be defined as a fractional expression with 100 as its denominator.

When we talk of 10 percentage of a number, we mean 10 parts put of one hundred parts of the number in consider action the word "percentage" can be denoted by the sign (%).

In the above example 10 percentages can be written as 10% or even$\frac{10}{100}$. When written in the form $\frac{10}{100}$, it is in a fraction form whereby the upper number is the numerator and the bottom the denominator. It can further be simplified as –

$$\frac{10}{100} = \frac{1}{10}$$

From the above discussion we can conclude that when dealing with percentage, a number can be expressed as a fraction of percentage, i.e.,
$\frac{10}{100} = \frac{1}{10}$; or it can be written just in percentage form, i.e., 10 percent = 10%.

Change fractions into percentage –
When changing a fraction into a percentage, we just multiply it by 100 and put the sign %.
Example: Express $\frac{1}{10}$ as a percentage = $\frac{1}{10} X \frac{100}{1} = 10\%$

Change percentage into fraction –
To change a percentage given into a fraction, we divide the fraction by 100.
Example: Express 10% as a fraction = $\frac{10}{10} = \frac{1}{10}$

To find percentage of quantity with another quantity –
Let x and y be two quantities of same type and rate percentage r, such that
$\qquad$ r % of x = y
$\qquad\qquad\qquad\qquad$ or
x.$\frac{r}{100} = y$
r=$\frac{y\ X\ 100}{x}$
i.e., Rate percent = $\frac{The\ quantity\ \ which\ represent\ \ in\ percent}{Second\ \ quantity} X\ 100$
Example: What percent Rs. 20 of Rs. 350?
Solution: r% $\frac{20 X 100}{350} = 5\frac{5}{7}$

To find the quantity when rate percent and percentage value are known –
If rate percent value are given then
Quantity = $\frac{Percent\ \ value\ X\ 100}{Rate\ \ percent}$

Chapter -3 – COMMISSION

The terms commission and discount are commonly applicable in the business world. We should clearly understand the terminologies before solving questions related with them.

Who is an Agent?
Usually businessman may not be directly doing the business transactions themselves because of expanded area of business. They may employ persons to be doing the selling or buying on their behalf. Such person are known as agents. Agents get commission against their works performance.

Commission –
Having transacted the business transactions, the agents will require remuneration from their principal such as remuneration is known as commission. Usually the commission is calculated on the basis of the percentage of total sales done by the agent.

Who is a Broker?
The buyer and seller may not come into contact face to face. Their transaction may be made possible by a middleman. He negotiates the sales and purchase proceeds between the buyer and seller such a negotiator is known as broker.

Brokerage –
This is the remuneration paid to the broker. It is actually a commission paid to the broker. It is calculated on the basis of percentage of the total value of the business transacted by the broker.

Del Credere Agent –
A del-credere agent is a person who guarantees collection of dues for the principal from the customers. They got a special type of commission known as del-credere commission. Usually they deduct the commission on the dues collected and remit the remaining amount to the principal.

Travelling Agent –
This is a person who moves round the trading zone of the principal doing the selling proceeds.

Important formulae –

i)   Amount of commission $= \dfrac{Rate\ of\ commission\ X\ Amount\ of\ sales}{100}$

ii)   Rate of commission $= \dfrac{Rate\ of\ commission\ X\ 100}{Amount\ of\ Sales}$

iii) Amount of Sales $= \dfrac{Rate\ of\ commission\ X\ 100}{Rate\ of\ commission}$

iv)   Amount of Del-cradere commission $= \dfrac{Credit\ Sales\ X\ Rate\ of\ del-credare\ commission}{100}$

DISCOUNT

The allowance or deduction from the market price of goods sold given by the vendor (Seller) to the purchaser (Buyer) is called discount. Discount is also known as allowance. The objective of allowing discount are –
- To increase the sales
- To retain the customership
- To encourage the customers to make the payment early

Kinds of Discount –
General there are two types of discounts are allowed to the customers –
Trade Discount and Cash Discount

1) **Trade Discount –** The Discount which is allowed by the seller according to the customs and traditions of the Business and which is allowed to all the customers irrespective of the payments conditions is called Trade Discount. The objective allowing Trade Discount is to increase the sales.

2) **Cash Discount –** The deduction on the marked price or invoice price or the selling price to the customer to encourage them to pay in cash or to make earlier cash payments is called cash discount.

In general Trade discount is given on marked price and cash discount is given on the remaining amount after deducting trade discount. In this way the purchaser in cash is entitled to get both type of discount.

Apart from these two discounts, there are some more types of discount.

Bulk discount or Quantity discount – It is allowed to the customers on purchasing on good in big quantity or bulk quantity.

Successive discount – When another discount is given after a discount, then the combination of these two discounts are known as successive discounts.

Equivalent Rate of Discount – The discount for which the amount due is equal o the amount due for successive discount is called their equivalent discount. Equivalent discount rate is also called single rate of equivalent discount.

It is to be noted that the total amount of successive discount is equal to the among of equivalent discount.

For example:
If a trader allows successive discount of 20% and 5% then the single rate/equivalent rate of discount will be –
D = 20+5 – 20x5 / 100 = 24%

NINE-VALUE TABLE
It is a method of calculating discount on a certain sum of list price/marked price. In this method on the basis of rate given first of all we have to calculate the discount for Rupee 1 and accordingly for Rupees 2, 3, 4, 5, 6, 7, 8 and 9.

With the help of this table we can calculate the commission or discount on any quantity.

Questions to be prepared on Unit-I :
1)   Give the definition and characteristics of Ratio and also explain its types.
2)   Describe the various type of Proportion.
3)   Distinguish between Ratio and Proportion.
4)   Explain the importance/significance of Percentage.
5)   Explain the terms Commission, Discount & Brokerage.
6)   What is successive discount?
7)   Explain equivalent rate of discount with example.
8)   Explain Nine-Values Table with example.

UNIT-II
Chapter – 4 - Simultaneous Equations

Equation – Equations signify relation of equality between two algebraic expressions symbolized by the sign of equality '='. In other words, an equation is statement which says that the two algebraic expressions are equal and is satisfied only for certain values of the variables.

Identify – When equality of two algebraic expressions hold true for all values of variables then it is called an identity.

Root of an Equation – The value of unknown or variable for which the equation is true is known as the root of the equation. To find the roots of an equation means to solve the equations.

Degree of an Equation – The degree of an equation is the highest exponent of the variable x or variables (x, y, …) present in the equation is called the degree of an equation.

Linear Equation – An equation which involves power of an unknown quantity not higher than unity (one) is called a linear equation.

One variable Linear Equation – A linear equation in one variable (x, say) in which the highest degree of the variable x is 1. A linear equation in one variable is, in general, written as ax+by = c or ax = c. This equation is also called, "First degree equation in x" or simple equation.

Two variable equation – A linear equation in two variables (x, y, say) in which the highest degree of the variables x and y each is 1. A linear equation in two variables, is general, is written as ax+by+c = 0 or ax+by=d.

Three variable equation – A linear equation in three variables (x, y, z, say) in which the highest degree of the variables x, y and z each is 1. A linear equation in three variables, in general, is written as $a_1x+b_1y+c_1z=d$.

Types of Simultaneous Equations –
    i)   Linear Simultaneous Equations in two Variables – Two linear equations in two variables together are linear simultaneous equations in two variables, e.g.:

$$4x+y = 2$$
$$3x-5y = 18$$

    ii)   Linear Simultaneous Equations in three Variables – Three linear equations in three variables together are linear simultaneous equations in three variables, e.g.:

$$3x+5y-7z = 13$$
$$4x+y-12z = 6$$
$$2x+9y-3z = 20$$

    iii)   Specific type of Simultaneous Equations – The equations in other than linear form are called specific type equations, e.g.:
        i)   quadratic equation : $ax^2 + bx + c = 0$
        ii) reciprocal equation : $\frac{a}{x} + \frac{b}{y} = c$
        iii) $a\left(\frac{y}{x}\right) + c = by, etc.$

Characteristics of Simultaneous Equations –
    1) A system of linear equations in one variable is not taken under simultaneous equations.

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2) The set of values of two variables x and y which satisfy each equation in the system of equations is called the solution of simultaneous equations.

The solutions of two variable linear simultaneous equations may be –

   i)   Infinitely many,
   ii)   An unique solution, or
   iii) No solution.

3) For simultaneous equations –

   $a_1x + b_1y = c_1$ and $a_2x + b_2y = c_2$

   a. If $\frac{a1}{a2} = \frac{b1}{b2} = k$ and $c1 = k$ C_2 then there are infinitely many solutions.

   b. If $\frac{a1}{a2} = \frac{b1}{b2} = c1 \neq kc_2$, then there is no solution.

   c. If $c_2 \neq 0$, then $c_1 = kc2 \rightarrow \frac{c1}{c2} = k, hence$

   $$\frac{a1}{a2} = \frac{b1}{b2} = \frac{c1}{c2} \rightarrow infinitely\ many\ solutions$$

   $$and\ \frac{a1}{a2} = \frac{b1}{b2} \neq \frac{c1}{c2} \rightarrow no\ solution$$

   d. If c_1 and c_2 both are zero (i.e., $c_1=0=c_2$)

Chapter – 5 - Preparation of Invoice

After the dispatch of goods, the seller prepares an invoice of the goods sold in which the quantity and quality of goods and their price is mentioned. Discount, if any, is deducted from the total amount, to this are added the seller's other expenses. Railway receipt number is also mentioned in the invoice if the goods have been sent by train.

Types of Invoice –

1) *Local invoices* – In these invoices only the cost of the goods less any trade discount is borne by the seller. All expenses of packing, cartage, loading and freight for carrying goods to the place of buyer are shown extra and charged to the buyer. It means that the cost includes cost of the goods only and all other expenses are extra and are recovered from the buyer.

2) *At station invoice* – This implies that all costs upto the stage of putting the goods at railway station will be borne by the seller and are included in the cost of goods but expenses beyond that, i.e., the railway fare, insurance, etc., will be borne by the buyer.

3) *Free on Rails (FOR) invoices* – Under this all the costs, i.e., cost of goods, cost of packing, carrying the goods to railway station, loading them in wagons, are borne by the seller and further expenses are borne by the buyer, i.e., they are charged over and above the cost of goods.

4) *Cost and Freight (C and F) invoice* – In such type of invoices it is presumed that cost which the seller is charging includes cost of goods, cost of packing, freight for carrying goods to the buyer and all other incidental expenses. All expenses other than above, i.e., insurance will be charged extra.

5) *Cost, Insurance and freight, (CI & F) invoice* – In such invoices cost charged by the seller includes cost of goods, cost of packing, freight and insurance. Other expenses are charged extra.

6) *Franco invoice* – In such invoices all costs upto putting the goods at the door of the buyer are borne by the seller, i.e., the price which he has quoted includes all expenses incurred in carrying the goods to the buyer's place. For example, Franco cost invoice will mean that cost charged by the seller includes cost of the goods, cost of packing, freight, insurance and local transportation charges.

Preparation of Invoice –

Invoice is prepared in duplicate. The original copy is sent to the buyer and the duplicate is kept for future reference. The usual contents of the invoice are –
   i)   Name and address of the seller
   ii)   No. and date of the invoice
   iii)   No. of the purchase order
   iv)   Name and address of the buyer
   v)   Place where it has been made
   vi)   Terms of trade
   vii)   Details about quantity of goods like weight or length etc.
   viii)   Date
   ix)   Separate price of each item and total price.
   x)   Trade discount, if any.
   xi)   Expenses incurred on sending the goods.
   xii)   Advance payment received (if paid by the buyer)
   xiii)   Net amount payable.
   xiv)   Details about mode of sending the goods
   xv)   Errors and omissions excepted
   xvi)   Special information, if any
   xvii)   Signature of the seller.

Uses of Invoice –
Following are the uses of invoice –
   a)   It informs the buyer about the price of the goods and other expenses he has to pay.
   b)   If the invoice reaches buyer before the goods, he can make arrangement for their resale.
   c)   The buyer can compare the invoice with his order.
   d)   After taking delivery of the goods he can compare the contents of the packages with the invoice and point discrepancy, if any to the seller.
   e)   Pay Octroi etc. on the basis of the invoice.

Necessary entries can be made in the books of accounts on the basis of invoice.

UNIT-III

Definition of Matrix –

A matrix (plural of matrices) is an array of real numbers (or other suitable elements) arranged in row and columns is called as a matrix. Consider a set of real numbers m and n when multiplied together we get mxn or mn. These can be used to define a matrix.

1) **Row Matrix or Row Vector –**
   A matrix having only one row is known as a row matrix or a row vector. It is in the form $(1xn)$.
   Example –

$$(3), \quad (7-10), \quad \left(\tfrac{1}{2} \ 3-6\right), \quad (a \ b \ c \ d)$$

2) **Column Matrix or Column Vector –**
   This is a type of Matrix which has only one column. It is in the form $(m \ x \ 1)$.
   Example –

$$(5), \quad \begin{bmatrix} 8 \\ 4 \\ -2 \end{bmatrix}, \quad \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix}$$

3) **Zero or Null Matrix –**
   This is a type of Matrix whose every element is zero. It is usually denoted by bold face zero **(0)**.
   Example –

   Examples :

$$(0), \quad \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}, \quad \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad [0 \ 0 \ 0 \ 0 \ 0]$$

4) **Diagonal Matrix –**
   Some matrix are such that all their elements are zero apart from the diagonal extending from the upper left hand corner to the lower right hand corner. These are known as diagonal matrix.
   Examples are -

$$\begin{bmatrix} 4 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}, \quad \begin{bmatrix} 3 & 0 \\ 0 & 7 \end{bmatrix}, \quad \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 7 & 0 & 0 \\ 0 & 0 & 9 & 0 \\ 0 & 0 & 0 & -2 \end{bmatrix}$$

5) **Square Matrix –**
   In this matrix, the number of rows and columns are the same.
   Examples –

$$(3), \quad \begin{bmatrix} 6 & -7 \\ 2 & 2 \end{bmatrix}, \quad \begin{bmatrix} 3 & 5 & 1/2 \\ 7 & 6 & 3 \\ 2 & 7 & -1 \end{bmatrix}, \quad \begin{bmatrix} 1/2 & 2 & 7 & -5 \\ 3 & 6 & 8 & 2 \\ 5 & 3/4 & -6 & 5 \\ 7 & 8 & -1 & 4 \end{bmatrix}$$

6) Unit or Identity Matrix –

This is a type of matrix where diagonal elements have values of 1. A unit matrix is usually denoted by bold face **(I)**. Examples of unit matrix are as follows –

$$I_1 = [\,1\,], \quad I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad I_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

7) Scalar Matrix –

This is a diagonal matrix whose diagonal elements are all equal. See examples given below –

$$\begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}, \quad \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}, \quad \begin{bmatrix} \sqrt{2} & 0 & 0 & 0 \\ 0 & \sqrt{2} & 0 & 0 \\ 0 & 0 & \sqrt{2} & 0 \\ 0 & 0 & 0 & \sqrt{2} \end{bmatrix}$$

8) Upper Triangular Matrix –

A square matrix in which every element below the principal diagonal are zero is known as an upper triangular matrix. Examples –

$$\begin{bmatrix} 6 & 2 & 4 \\ 0 & 5 & 7 \\ 0 & 0 & 3 \end{bmatrix}, \quad \begin{bmatrix} 7 & 5 & 4 & 6 \\ 0 & 8 & 3 & 7 \\ 0 & 0 & 2 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

9) Lower Triangular Matrix –

A square matrix in which every element above the principal diagonal are zero is known as the lower triangular matrix. Examples –

$$\begin{bmatrix} 6 & 0 & 0 \\ 2 & 4 & 0 \\ 5 & 3 & 5 \end{bmatrix}, \quad \begin{bmatrix} 8 & 0 & 0 & 0 \\ 4 & 7 & 0 & 0 \\ 3 & 4 & 2 & 0 \\ 9 & 3 & 6 & 5 \end{bmatrix}$$

10) Transpose Matrix –

A matrix obtained by interchanging the row and columns of a matrix is called transpose of A and is denoted by A^T or A'. Example given below –

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \quad A_T = A' = \begin{bmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{bmatrix}$$

$$A = \begin{bmatrix} 7 & 6 & 4 \\ 8 & 5 & 3 \\ 9 & 2 & 1 \end{bmatrix} \quad A_T = A' = \begin{bmatrix} 7 & 8 & 9 \\ 6 & 5 & 2 \\ 4 & 3 & 1 \end{bmatrix}$$

UNIT-IV

FUNDAMENTAL LAWS OF LOGARITHM

The laws of logarithm which are valid for any base a (> 0 but $\neq 1$) are as follows:

Law 1 : The logarithm of the product of two numbers is equal to the sum of the logarithms of these numbers to the same base, i.e.,

$$\log_a(m \times n) = \log_a m + \log_a n$$

Proof : Let $\qquad \log_a m = x \quad \Rightarrow \quad a^x = m$

and $\qquad\qquad \log_a n = y \quad \Rightarrow \quad a^y = n$

Hence $\qquad\qquad a^x \times a^y = m \times n$

or $\qquad\qquad a^{x+y} = m \times n$

$\therefore \qquad\qquad \log_a(m \times n) = x + y,$ $\qquad\qquad\qquad$ by definition

$\Rightarrow \qquad\qquad \log_a(m \times n) = \log_a m + \log_a n.$

Sub-law : The logarithm of the product of two or more numbers is equal to the sum of the logarithms of these numbers to the same base, i.e.,

$$\log_a(m \times n \times p \times .....) = \log_a m + \log_a n + \log_a p + ...$$

Law 2 : The logarithm of the quotient of two numbers is equal to the difference of the logarithm of the numerator and the logarithm of the denominator to the same base, i.e.,

$$\log_a\left(\frac{m}{n}\right) = \log_a m - \log_a n$$

Proof : Let $\qquad \log_a m = x \quad \Rightarrow \quad a^x = m$

and $\qquad\qquad \log_a n = y \quad \Rightarrow \quad a^y = n$

Hence $\qquad\qquad \dfrac{m}{n} = \dfrac{a^x}{a^y} = a^{x-y},$ $\qquad\qquad$ by law of indices

$\Rightarrow \qquad\qquad \log_a\left(\dfrac{m}{n}\right) = x - y$

$\Rightarrow \qquad\qquad \log_a\left(\dfrac{m}{n}\right) = \log_a m - \log_a n$

Law 3 : The logarithm of a number raised to a power is equal to the product of the index of the power and the logarithm of the number to the same base, i.e.,

$$\log_a(m^n) = n \log_a m$$

Proof : Let $\qquad \log_a m = x \quad \Rightarrow \quad a^x = m$

Hence, $\qquad\qquad (a^x)^n = m^n,$ by raising to the power n on both sides.

$\Rightarrow \qquad\qquad a^{nx} = m^n$

$\Rightarrow \qquad\qquad \log_a(m^n) = nx = n \log_a m.$

Remark : If $n = \dfrac{1}{p}$, where p is a positive integer, then logarithm $\sqrt[p]{m}$ or $m^{1/p}$ is obtained by dividing the logarithm of m by p, i.e.,

$$\log_a(\sqrt[p]{m}) = \log_a(m^{1/p}) = \frac{1}{p}\log_a m \text{ or } \frac{\log_a m}{p}$$

Change of Base

If the logarithm of numbers are given to the base a, then the logarithms of those numbers can be obtained to another base b.

Let $\qquad\qquad \log_b n = x$

$\Rightarrow \qquad\qquad b^x = n$

$\Rightarrow \qquad\qquad \log_a b^x = \log_a n,$ $\qquad\qquad$ {taking log to the base a}

$\Rightarrow \qquad\qquad x \log_a b = \log_a n$

$\Rightarrow \qquad\qquad x = \dfrac{\log_a n}{\log_a b}$ $\quad$ or $\quad$ $\log_b n = \dfrac{\log_a n}{\log_a b}$ $\qquad$ ...(1)

Writing $n = a$ in (1),

$$\log_b a = \frac{\log_a a}{\log_a b} = \frac{1}{\log_a b}$$

$\Rightarrow \qquad\qquad \log_b a . \log_a b = 1 \qquad\qquad\qquad ...(2)$

Similarly $\qquad\qquad \log_b m = n$

$\Rightarrow \qquad\qquad\qquad b^n = m$

$\Rightarrow \qquad\qquad\qquad \left(\frac{1}{b}\right)^{-n} = m$

$\Rightarrow \qquad\qquad \log_{1/b} m = -n \qquad\qquad\qquad ...(3)$

Common Logarithm*

The logarithm of a number to the base 10 is called 'common logarithm'. In numerical calculations common logarithms are used when no base is mentioned it is understood to be 10. For example,

$$\log_{10} n = x \text{ is simply written as } \log n = x.$$

The common logarithm of a number consists of two parts : the whole part or the integral part and the decimal part. The integral part is called *characteristic* and the decimal part is called *mantissa*.

$$\text{In } \log_{10} 14 = 1.1461 = 1 + .1461$$

characteristic is 1 and mantissa is .1416.

To Determine Characteristic of a Common Logarithm

Consider the logarithm of those numbers which are integral powers of 10.

$$10^3 = 1,000 \quad\Rightarrow\quad \log_{10} 1,000 = 3$$
$$10^2 = 100 \quad\Rightarrow\quad \log_{10} 100 = 2$$
$$10^1 = 10 \quad\Rightarrow\quad \log_{10} 10 = 1$$
$$10^0 = 1 \quad\Rightarrow\quad \log_{10} 1 = 0$$
$$10^{-\infty} = 0 \quad\Rightarrow\quad \log_{10} 0 = -\infty$$
$$10^{-1} = 0.1 \quad\Rightarrow\quad \log_{10} 0.1 = -1$$
$$10^{-2} = 0.01 \quad\Rightarrow\quad \log_{10} 0.01 = -2$$
$$10^{-3} = 0.001 \quad\Rightarrow\quad \log_{10} 0.001 = -3$$

From the above results we observe that :

(1) The numbers which are integral powers of 10 are straight forward and it contains characteristic only.

(2) The common logarithm of a number between 1 and 10 lies between 0 and 1.

(3) The common logarithm of a number between 10 and 100 lies between 1 and 2.

(4) The common logarithm of a number between 100 and 1,000 lies between 2 and 3.

(5) The common logarithm of a number between .1 and 1 is between −1 and 0.

(6) The common logarithm of a number between .01 and 0.1 is between (−2) and (−1).

(7) The characteristic of a common logarithm is found out by mere inspection. In this connection the following rules should be kept in mind.

SIMPLE INTEREST

Interest – Interest is the money which the lender of a sum receives from the borrower, in consideration of the borrower using the sum.

Usually money is given out by lenders and the borrowers have to pay interest when it falls due. When it is said on this basis, we call it Simple Interest.

Definitions of Usual Words –
1) *Interest* – If a businessman borrows some money from a bank and in consideration for the use of money so borrowed, he pays the bank something apart from the principal amount, this consideration paid is interest.
2) *Principal Amount* – This is the amount which is given as credit to the borrower. It is the lent out sum of money.
3) *Amount* – When the interest has been added to be principal amount, the total is known as amount.
4) *Time* – This is the duration in which the principal amount has been taken by the borrower. It may be monthly, quarterly, half yearly, yearly etc. Interest is calculated on the basis of time.
5) *Rate of Interest* – This is the interest charged for one unit of principal for a specific period, e.g. 1 year.

Formula for the Calculation of Simple Interest –
When calculating simple interest, we take into account the following terms –
1) Principal   -   denoted as P
2) Rate of interest-   denoted as R
3) Interest   -   denoted as I
4) Amount   -   denoted as A
5) Time   -   denoted as T

i)   When calculating **Interest**,
$$I = \frac{P x R x T}{100}$$
ii)   When calculating **Time**,
$$T = \frac{I x 100}{P \ x \ R}$$
iii)   When calculating **Principal**,
$$P = \frac{I x 100}{R \ x \ T}$$
iv)   When calculating **Rate**,
$$R = \frac{I x 100}{P \ x \ T}$$
v)   When calculating **Amount**,
$$A = P + \frac{P \ x \ R \ x \ T}{100}$$
or $$A = P \ x \left(1 + \frac{R \ x \ T}{100}\right)$$

or   A = P + I

Meaning of Compound Interest –
By compound interest we mean when interest becomes due after a certain period, it is added to the principal amount and interest on succeeding years is based on the principal and the interest added. The difference between the amount and the original principal is called the compound interest.

It means that in compound interest, the principal doesn't remain fixed at the original sum but increase at the end of each interest period. Interest period is the period at which the interest becomes due. It may be a year, half year or quarter year.

Distinction between Simple and Compound Interest –

S.No.	Basis of Difference	Simple Interest	Compound Interest
1	Principal and Interest	Calculated on original amount	Calculated with interest included
2	calculation of interest	Calculated on overall period in single time.	Calculated yearly, half yearly, quarterly or even monthly.
3	Principal Amount	Always same on whole period.	Changes from year to year, month to month etc.
4	Difference in amount of interest	Less than that of compound interest.	It is more than that of small interest.

Methods for Calculation of Compound Interest –
The following are some of the methods used to calculate compound interest –
   1) Simple interest method.
   2) Interest table method.
   3) Decimal point method.
   4) Compound interest formula method.
   5) By Logarithm method.

1) Simple Interest Method –
        When the time of the interest is not so long, i.e.; when interest is calculated for only a few years then we use this method. It is just similar to that used to find out simple interest. Follow the steps below –
   i)   Calculate interest on principal at the end of every year.
   ii)   Add the interest got in step (i) above to the original principal. This amount is principal for the next year.
   iii)   Calculate compound interest by adding each year's interest for the entire period.
   iv)   Finally subtract the original from the compounded amount and this gives the compound interest.

2) Compound Interest Formula Method –
        When the number of year involved to calculate the compound interest are many, we use the above method. The formula used is –
$$A = P\left(1 + \frac{R}{100}\right)^n$$

Where P denotes   = Principal (original)
                n = number of years (interest period)
                r   = rate of interest (in percentage)
                A   = Amount after n years.

MEANING IMPORTANT FORMULAE RELATED WITH PROFIT AND LOSS
When we are given cost price and selling price, we can formulate some formula related with them. We can abbreviate the two as below –
Cost Price   =   C.P.
Selling Price   =   S.P.

1)   Gain % on cost   = $\dfrac{S.P.-C.P.}{C.P.} \times 100$

                                                  $\dfrac{Gain \times 100}{Cost\ Price}$

2)   Loss % on cost   = $\dfrac{C.P.-S.P.}{C.P.} \times 100$

                                                  $\dfrac{Net\ Loss}{C.P.} \times 100$

3)   Gain % on sales   = $\dfrac{S.P.-C.P.}{S.P.} \times 100$

                                                  $\dfrac{Gain}{S.P.} \times 100$

4)   Loss % on sales   = $\dfrac{C.P.-S.P.}{S.P.} \times 100$

                                                  $\dfrac{Loss}{S.P.} \times 100$

UNIT-V
AVERAGE

"Average is the most popular medium to express the normal or general tendency of a group of figures. This is more useful specifically in the group where equal weitage is to be given to all the items of the group. It is more useful to study and analysis the social and economic problems. We use to word average in our day to day life e.g. average salary, average consumption, average production. Average cost, average rainfall, average produce of crops etc.

Object and Function of Average

1) *Compact and easy representation:* First and foremost work of average is to represent scattered complicated data into single simple and easy form.
2) *Provide base for comparison:* With the help of average we can easily compare the important features of any two different group.
3) *Single representation:* Average gives a bird's eye view of the huge mass of statistical data which ordinarily are not easily understandable. It grasps the central theme of a data which are reduced to single figure.
4) *Base of Statistical analysis:* It provides a sound base for further statistical analysis of data. It is used in all the statistical tools.
5) *Useful for Planning:* It is more useful for future planning.

Disadvantages or limitations of average

1) It may not be represented in actual data and so it is theoretical.
2) The extreme values have greater effect on mean.
3) It cannot be calculated for the qualitative data such as anger, love, honesty etc.
4) Mean may lead to fallacious conditions in the absence of original observation.

STATISTICAL AVERAGES

The point around which the observations concentrate in general in the central part of the data is called central value of the data and the tendency of the observations to concentrate around a central point is known as Central Tendency.

Objects of Statistical Average:

➢ To get a single value that describes the characteristics of the entire group
➢ To facilitate comparison

Functions of Statistical Average:

➢ Gives information about the whole group
➢ Becomes the basis of future planning and actions
➢ Provides a basis for analysis
➢ Traces mathematical relationships
➢ Helps in decision making

Requisites of an Ideal Average:

➢ Simple and rigid definition
➢ Easy to understand
➢ Simple and easy to compute
➢ Based on all observations
➢ Least affected by extreme values
➢ Least affected by fluctuations of sampling
➢ Capable of further algebric treatment

ARITHMETIC MEAN ($\overline{X}$)

Arithmetic Mean of a group of observations is the quotient obtained by dividing the sum of all observations by their number. It is the most commonly used average or measure of the central tendency applicable only in case of quantitative data. Arithmetic mean is also simply called "mean".

Arithmetic mean is denoted by $\overline{X}$.

Merits of Arithmetic Mean:
- It is rigidly defined.
- It is easy to calculate and simple to follow.
- It is based on all the observations.
- It is readily put to algebraic treatment.
- It is least affected by fluctuations of sampling.
- It is not necessary to arrange the data in ascending or descending order.

Demerits of Arithmetic Mean:
- The arithmetic mean is highly affected by extreme values.
- It cannot average the ratios and percentages properly.
- It cannot be computed accurately if any item is missing.
- The mean sometimes does not coincide with any of the observed value.
- It cannot be determined by inspection.
- It cannot be calculated in case of open ended classes.

Methods of Calculating Arithmetic Mean:
- Direct Method
- Short cut method
- Step deviation method

Use of Arithmetic Mean:
Arithmetic Mean is recommended in following situation:
- When the frequency distribution is symmetrical.
- When we need a stable average.
- When other measures such as standard deviation, coefficient of correlation are to be computed later.

MEDIAN (M)

The median is that value of the variable which divides the group into two equal parts, one part comprising of all values greater and other of all values less than the median. For calculation of median the data has to be arranged in either ascending or descending order. Median is denoted by **M**.

Merits of Median:
- It is easily understood and easy to calculate.
- It is rigidly defined.
- It can sometimes be located by simple inspection and can also be computed graphically.
- It is positional average therefore not affected at all by extreme observations.
- It is only average to be used while dealing with qualitative data like intelligence, honesty etc.
- It is especially useful in case of open end classes since only the position and not the value of items must be known.
- It is not affected by extreme values.

Demerits of Median:
- For calculation, it is necessary to arrange data in ascending or descending order.
- Since it is a positional average, its value is not determined by each and every observation.
- It is not suitable for further algebric treatment.
- It is not accurate for large data.
- The value of median is more affected by sampling fluctuations than the value of the arithmetic mean.

Uses of Median:
The use of median is recommended in the following situations:
- When there are open-ended classes provided it does not fall in those classes.
- When exceptionally large or small values occur at the ends of the frequency distribution.
- When the observation cannot be measured numerically but can be ranked in order.
- To determine the typical value in the problems concerning distribution of wealth etc.

MODE (Z)
Mode is the value which occurs the greatest number of times in the data. The word mode has been derived from the French word **'La Mode'** which implies fashion. The Mode of a distribution is the value at the point around which the items tend to be most heavily concentrated. It may be regarded as the most typical of a series of values. Mode is denoted by **Z.**

Merits of Mode:
- It is easy to understand and simple to calculate.
- It is not affected by extreme large or small values.
- It can be located only by inspection in ungrouped data and discrete frequency distribution.
- It can be useful for qualitative data.
- It can be computed in open-end frequency table.
- It can be located graphically.

Demerits of Mode:
- It is not well defined.
- It is not based on all the values.
- It is suitable for large values and it will not be well defined if the data consists of small number of values.
- It is not capable of further mathematical treatment.
- Sometimes, the data has one or more than one mode and sometimes the data has no mode at all.

Uses of Mode:
The use of mode is recommended in the following situations:
- When a quick approximate measure of central tendency is desired.
- When the measure of central tendency should be the most typical value.

GEOMETRIC MEAN (G.M)
The geometric mean also called geometric average is the nth root of the product of n non-negative quantities. Geometric Mean is denoted by **G.M**.

Properties of Geometric Mean:
- The geometric mean is less than arithmetic mean, G.M<A.M
- The product of the items remains unchanged if each item is replaced by the geometric mean.

- The geometric mean of the ratio of corresponding observations in two series is equal to the ratios their geometric means.
- The geometric mean of the products of corresponding items in two series.

Merits of Geometric Mean:
- It is rigidly defined and its value is a precise figure.
- It is based on all observations.
- It is capable of further algebraic treatment.
- It is not much affected by fluctuation of sampling.
- It is not affected by extreme values.

Demerits of Geometric Mean:
- It cannot be calculated if any of the observation is zero or negative.
- Its calculation is rather difficult.
- It is not easy to understand.
- It may not coincide with any of the observations.

Uses of Geometric Mean:
- Geometric Mean is appropriate when:
  - Large observations are to be given less weight.
  - We find the relative changes such as the average rate of population growth, the average rate of intrest etc.
  - Where some of the observations are too small and/or too large.
- Also used for construction of Index Numbers.

HARMONIC MEAN (H.M)
Harmonic mean is another measure of central tendency. Harmonic mean is also useful for quantitative data. *Harmonic mean is quotient of "number of the given values" and "sum of the reciprocals of the given values". It is denoted by **H.M.***

Merits of Harmonic Mean:
- It is based on all observations.
- It not much affected by the fluctuation of sampling.
- It is capable of algebraic treatment.
- It is an appropriate average for averaging ratios and rates.
- It does not give much weight to the large items and gives greater importance to small items.

Demerits of Harmonic Mean:
- Its calculation is difficult.
- It gives high weight-age to the small items.
- It cannot be calculated if any one of the items is zero.
- It is usually a value which does not exist in the given data.

Uses of Harmonic Mean:
- Harmonic mean is better in computation of average speed, average price etc. under certain conditions.

PROFIT & LOSS
Profit: Profit is the percentage returns on investment of capital. In other words it is the reward of ownership or reward of risk taking. When an article is sold for more than its cost or purchase price then the difference amount will be profit.

Profit = Selling Price – Purchase Price (Cost Price)

Loss: When an article is sold for less than its cost or purchase price difference is called loss.

Loss = Cost Price (Purchase Price) – Selling Price

When we are given cost price and selling price, we can formulate some formula related with them. We can abbreviate the two as below –

Cost Price   =   C.P.

Selling Price   =   S.P.

1) Gain % on cost   = $\dfrac{S.P. - C.P.}{C.P.} \, x \, 100$

$$\dfrac{Gain \, x \, 100}{Cost \, Price}$$

2) Loss % on cost   = $\dfrac{C.P. - S.P.}{C.P.} \, x \, 100$

$$\dfrac{Net \, Loss}{C.P.} \, x \, 100$$

3) Gain % on sales   = $\dfrac{S.P. - C.P.}{S.P.} \, x \, 100$

$$\dfrac{Gain}{S.P.} \, x \, 100$$

4) Loss % on sales   = $\dfrac{C.P. - S.P.}{S.P.} \, x \, 100$

$$\dfrac{Loss}{S.P.} \, x \, 100$$