SYLLABUS

Class – B.Com. I Year (Hons)

Subject :Business Mathematics

UNIT – I	Average, Ratio and Proportion, Percentage
UNIT – II	Profit and Loss, Simple Interest, Compound Interest
UNIT – III	Annuities, True Discount, Banker's Discount
UNIT – IV	Basic Concepts of Set Theory: Definition, Types, Operations on Sets, Venn Diagram. Simultaneous Equations: Meaning, Characteristics, Types and Calculations
UNIT – V	Quadratic Equation in one variable, inequalities, line programming (Two variable)

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UNIT-I
AVERAGE

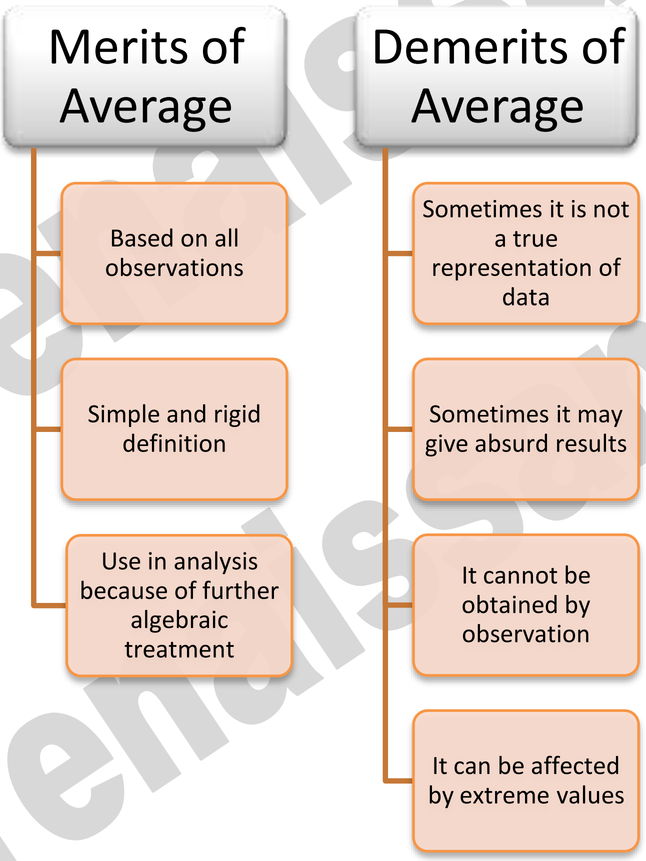
The average of the number of quantities of observations of the same kind is their sum divided by their number. The average is also called average value or mean value or arithmetic mean.

$$\text{Average} = \frac{\text{Sum of all Elements}}{\text{Total No. of Elements}}$$

$$\text{Average} = \frac{x_1 + x_2 + x_3 + \dots\dots\dots\dots + x_n}{n}$$, for observations x1 ,x2, x3, ...xn

Functions of Average

a) To present the salient features of data in simple and summarized form
b) To compare and draw conclusion
c) To get a simple value that describes the characteristics of the entire group
d) To help in statistical analysis



RATIO

A ratio can exist only between two quantities of the same type. If x and y are any two numbers and y $\neq 0$ then the fraction $\frac{x}{y}$ is called the ratio of x and y is written as x:y.

Characteristics of Ratio –

The following characteristics are attributed to ratio relationship:

i) Ratio is a cross relation found between two or more quantities of same type.

ii) It must be expressed in the same units.

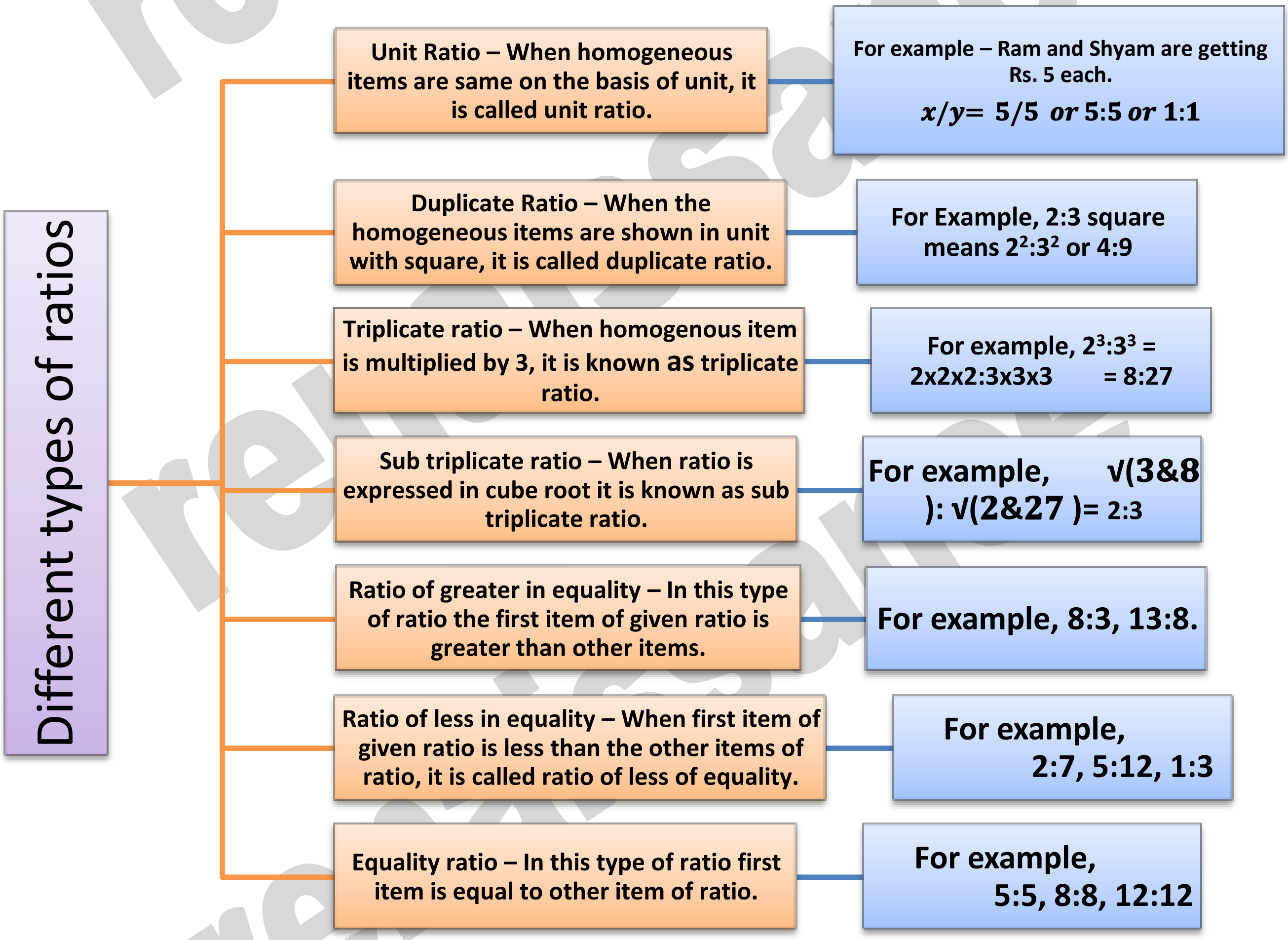
iii) By the fraction laws a ratio can be expressed as below:

$$\frac{y}{x} = \text{y: x}$$

$$\frac{10}{5} = 10:5 \text{ or } 2:1$$

iv) A ratio expresses the number of times that one quantity contains another.

v) Two or more ratios may be compared by reducing their equivalent fractions to a common denominator.



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PROPORTION

Relationship between the two ratio's is called proportion. Here, quantity ratio of first two items is equality to rest two terms.

For example, 2:5::6:15

Proportion is expressed by four parallel points (::).

In the simple proportion here its not necessary that two items of first ratio and the items of second ratio should be homogeneous. But the items of second set of ratio have the same relationship which is found between the items of first ratio. For example 2:5::6:15. Here 5 is 2.5 times of 2 in case of first ratio. In the same 15 is 2.5 times of 6 in the second set of ratio.

Characteristics of Proportion –

i) Proportion is given in four parts. So first number is known as first item, second number is second item, third number is third item and fourth number is known as fourth item.

ii) First and fourth items are known as extremes items and second and third items are known as mean items.

iii) It is not necessary in proportion that all four items should be homogenous. But the ratios of first and second and third and fourth should be the same.

Direct Proportion –

In this type of ratio, two different items has the such relation that if the one is increased or decreased, another will change accordingly in the same ratio.

Inverse Proportion – In this type of ratio, two different items has such a relation that if the one is increased or decreased, another will change inthe opposite direction in the same ratio.

Types of Proportion

Continued proportion –

If ratio of items is going on continuously, e.g., ratio of first and two is equal to two and three and ratio of two and three is equal to three and fourth item and so on, thus, ratio is known as continued ratio.

For example, A/B= B/C= C/D= D/E= E/F….

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Difference Between Ratio and Proportion –

S.No.	Ratio	Proportion
1	There are two terms in a ratio.	There are four terms in a proportion.
2	Comparison of two quantities of same type.	Comparison of two ratios.
3	Two quantities must be of same type.	All four quantities are not of same type but the first two are of one type and the last two may be of another type.
4	There is not a product rule	The product of extremes is equal to product of the means.

PERCENTAGE

Percent and Percentage

When we talk of percentage, we usually refer to "for every one hundred."
Actually percentage can be defined as a fractional expression with 100 as its denominator.

When we talk of 10 percentage of a number, we mean 10 parts put of one hundred parts of the number in consider action the word "percentage" can be denoted by the sign (%).

In the above example 10 percentages can be written as 10% or even $\frac{10}{100}$. When written in the form $\frac{10}{100}$, it is in a fraction form whereby the upper number is the numerator and the bottom the denominator. It can further be simplified as –

$$\frac{10}{100} = \frac{1}{10}$$

From the above discussion we can conclude that when dealing with percentage, a number can be expressed as a fraction of percentage, i.e.,
$\frac{10}{100} = \frac{1}{10}$; or it can be written just in percentage form, i.e., 10 percent = 10%.

Change fractions into percentage

When changing a fraction into a percentage, we just multiply it by 100 and put the sign %.

Example: Express 1/10 as a percentage = 1/10 X 100/1=10%

Change percentage into fraction

To change a percentage given into a fraction, we divide the fraction by 100

Example: Express 10% as a fraction = 10/10=1/10

To find percentage of quantity with another quantity –

Let x and y be two quantities of same type and rate percentage r, such that

$r\,\%\ of\ x = y$

or

$x.\frac{r}{100} = y$

$r = \frac{yX\ 100}{x}$

i.e., Rate percent = $\frac{The quantity which represent in percent}{Second quantity} X\ 100$

Example: What percent Rs. 20 of Rs. 350?

Solution: $r = \frac{20X100}{350} = 5\frac{5}{7}$

To find the quantity when rate percent and percentage value are known –

If rate percent value are given then

Quantity = $\frac{Percent value X\ \ 100}{Rate percent}$

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UNIT-II
SIMPLE INTEREST

Interest –Whenever we borrow a certain sum of money (known as the principal), we pay back the original amount accompanied with a certain amount of interest on that amount. In a way, those are the charges of borrowing that sum of money.

Simple interest is one method of determining the amount due at the end of loan duration.

Definitions of Usual Words –

Principal (P): The original sum of money loaned/deposited.

Interest (I): The amount of money that you pay to borrow money or the amount of money that you earn on a deposit.

Time (T): The duration for which the money is borrowed/deposited.

Rate of Interest (R): The percent of interest that you pay for money borrowed, or earn for money deposited

$$Simple\ Interest\ (SI) = \frac{P \times R \times T}{100}$$

Where:
P: Principal (original amount)
R: Rate of Interest (in %)
T: Time period (yearly, half-yearly etc.)

Amount Due at the end of the time period, **A = P (original amount) + SI**

$$A = P + \left\{ \frac{P \times R \times T}{100} \right\}$$

If you have a close look, Simple Interest is nothing else but an application of the concept of percentages.

Meaning of Compound Interest –
By compound interest we mean when interest becomes due after a certain period, it is added to the principal amount and interest on succeeding years is based on the principal and the interest added. The difference between the amount and the original principal is called the compound interest.

It means that in compound interest, the principal doesn't remain fixed at the original sum but increase at the end of each interest period. Interest period is the period at which the interest becomes due. It may be a year, half year or quarter year.

Methods for Calculation of Compound Interest –

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Basis of Difference	Simple Interest	Compound Interest
Principal and Interest	Calculated on original amount	Calculated with interest included
calculation of interest	Calculated on overall period in single time.	Calculated yearly, half yearly, quarterly or even monthly.
Principal Amount	Always same on whole period.	Changes from year to year, month to month etc.
Difference in amount of interest	Less than that of compound interest.	It is more than that of small interest.

The following are some of the methods used to calculate compound interest –

1) Simple interest method.
2) Interest table method.
3) Decimal point method.
4) Compound interest formula method.
5) By Logarithm method.

1) Simple Interest Method –

When the time of the interest is not so long, i.e.; when interest is calculated for only a few years then we use this method. It is just similar to that used to find out simple interest. Follow the steps below –

i) Calculate interest on principal at the end of every year.
ii) Add the interest got in step (i) above to the original principal. This amount is principal for the next year.
iii) Calculate compound interest by adding each year's interest for the entire period.
iv) Finally subtract the original from the compounded amount and this gives the compound interest.

2) Compound Interest Formula Method –

When the number of years involved to calculate the compound interest are many, we use the above method. The formula used is –

$$A = P\left(1 + \frac{R}{100}\right)^n$$

Where P denotes = Principal (original)
n = number of years (interest period)
r = rate of interest (in percentage)
A = Amount after n years.

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PROFIT AND LOSS

SOME IMPORTANT DEFINITIONSRELATED WITH PROFIT AND LOSS

Cost Price (CP)

The price, which is paid to acquire a product, is called cost price. All the overhead expenses (transportation, taxes etc.) are also included in the cost price.

Selling Price (SP)

The sum of money, which is finally received for the product i.e. the price at which the product is finally disposed off is called the Selling price.

Marked Price (MP)

The price, which is listed or marked on the product, is also known as quotation price/printed price/catalogue price/invoice price.

Profit

If selling price is greater than Cost price, then excess of SP to CP is called Gain or Profit.
PROFIT = SELLING PRICE – COST PRICE

Loss

If selling price is less than Cost price, then excess of CP to SP is called Loss.
LOSS = COST PRICE – SELLING PRICE

Profit percentage formula

Profit % = 100 × Profit/Cost Price.

Percentage Loss

Loss % = 100 × Loss/Cost Price.

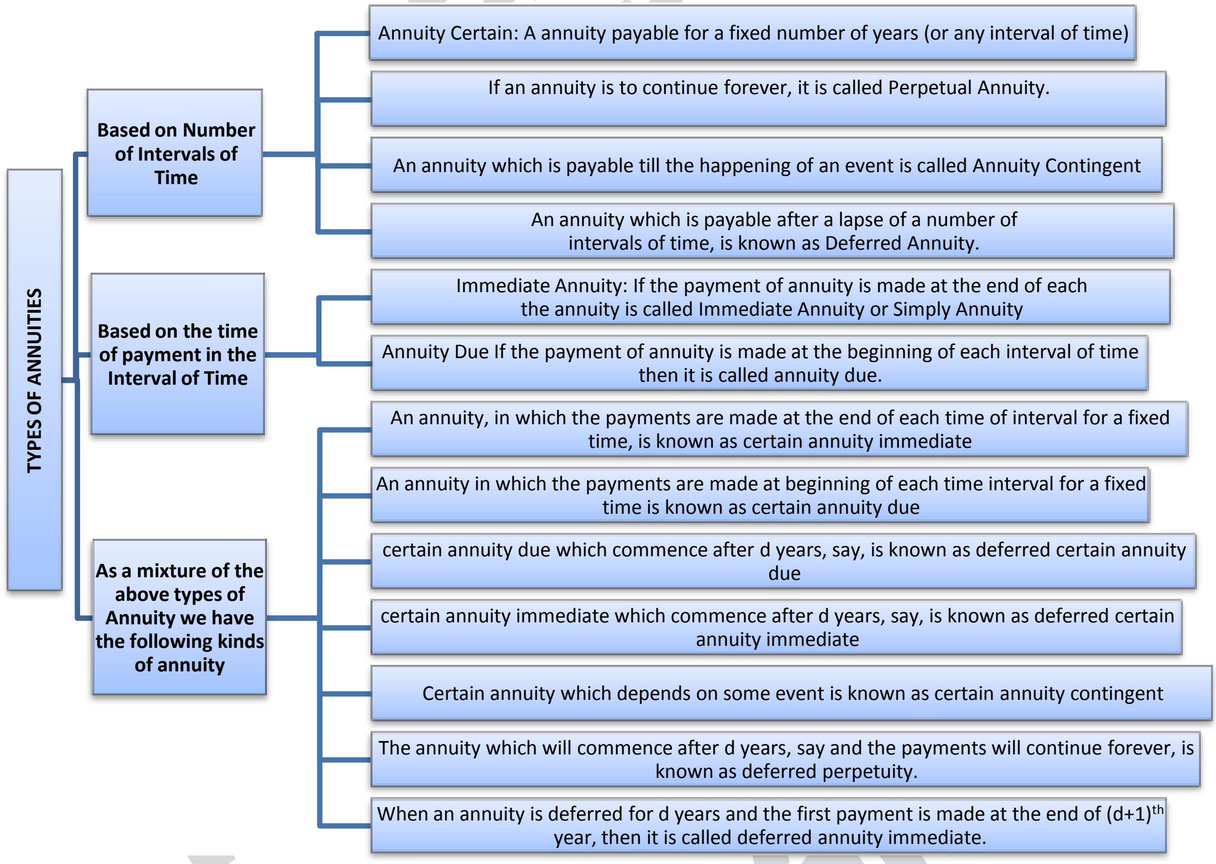
UNIT - III

Annuity, True Discount & Banker's Discount

Present worth/ Value: The money to be paid before due date to clear off a debt is called present worth.

MEANING AND DEFINITION OF ANNUITY

An annuity is fixed sum paid at regular intervals of time and a series of a fixed amount of money paid at equal intervals of time under certain conditions.
E.g. Payment of loan in installments, the payments of the price of goods purchased on installments, awarding scholarship
The interval of time may either a year or a half year or a quarter year or month



True Discount: The difference between the amount and the present worth is called the true discount (T.D.).Also; it is the interest on the present worth (P.W.) for the time which will elapse before the debt is due to be discharged

Banker's Discount : Assume that a merchant A purchases goods worth, say Rs.1000 from another merchant B at a credit of say 4 months.

Then B prepares a bill called bill of exchange (also called Hundi). On receipts of goods, A gives an agreement by signing on the bill allowing B to withdraw the money from A's bank exactly after 4 months of the date of the bill

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Main Symbols and Formula Based on Compound Interest

A = Amount of an Annuity
P = Present Value of an Annuity
a = Annuity or Instalment of an Annuity
i = Interest of Rupee one in a year, say
n = Number of years (or time intervals)

(i) Amount of Certain Annuity Immediate :

$$A = \frac{a}{i}\{(1+i)^n - 1\} \text{ or } A = P(1+i)^n$$

(ii) Present value of Certain Annuity ~~Due~~ :Immediate:

$$P = \frac{a}{i}\{1-(1+i)^{-n}\} \text{ or } \frac{a}{i}\left\{1-\frac{1}{(1+i)^n}\right\} \text{ or } \frac{A}{(1+i)^n}$$

(iii) Amount of Certain Annuity Due

$$A = \frac{a(1+i)}{i}\{(1+i)^n - 1\}$$

(iv) Present Value of Certain Annuity Due

$$P = \frac{a(1+i)}{i}\{1-(1+i)^{-n}\} \text{ or } \frac{a(1+i)}{i}\left\{1-\frac{1}{(1+i)^n}\right\}$$

(v) Present Value of Perpetuity :

$$P = \frac{a}{i}$$

(vi) Present Value of Deferred Certain Annuity for d years :

$$P = \frac{1}{(1+i)^d}\cdot\frac{a}{i}\left[1-\frac{1}{(1+i)^n}\right]$$

(vii) Present Value of Deferred Perpetuity for d years :

$$P = \frac{1}{(1+i)^d}\cdot\frac{a}{i}$$

The date exactly after 4 months is known as **nominally due date**. Three more days (called grace days) are added to this date to get a date known as **legally due date.**

The amount given on the bill is called the **Face Value (F)** which is Rs.1000 in this case.

Assume that B needs this money before the legally due date. He can approach a banker or broker who pays him the money against the bill, but somewhat less than the face value. The banker deducts the simple interest on the face value for the unexpired time. This deduction is known as **Bankers Discount (BD).** In another words, Bank Discount (BD) is the simple interest on the face value for the period from the date on which the bill was discounted and the legally due date.

The **present value** is the amount which, if placed at a particular rate for a specified period will amount to that sum of money at the end of the specified period. The interest on the present value is called the **True Discount (TD).** If the banker deducts the true discount on the face value for the unexpired time, he will not gain anything.

Banker's Gain (BG) is the difference between banker's discount and the true discount for the unexpired time.

Important Formulas - Banker's Discount

Let F = Face Value of the Bill, TD = True Discount, BD = Bankers Discount, BG = Banker's Gain, R = Rate of Interest, PW = Present Worth and T = Time in Years

$$BD = \text{Simple Interest on the face value of the bill for unexpired time} = \frac{FTR}{100}$$

$$PW = \frac{F}{1 + T\left(\frac{R}{100}\right)}$$

$$TD = \text{Simple Interest on the present value for unexpired time} = \frac{PW \times TR}{100} = \frac{FTR}{100 + TR}$$

$$TD = \frac{BD \times 100}{100 + TR}$$

$$PW = F - TD$$

$$F = \frac{BD \times TD}{BD - TD}$$

$$BG = BD - TD = \text{Simple Interest on TD} = \frac{(TD)^2}{PW}$$

TD

$= \sqrt{\boxed{PW \times BG}}$

TD = (BG×100)/TR

Unit – IV

Simultaneous Equations

Equation – Equations signify relation of equality between two algebraic expressions symbolized by the sign of equality '='. In other words, an equation is statement which says that the two algebraic expressions are equal and is satisfied only for certain values of the variables.

Identify – When equality of two algebraic expressions hold true for all values of variables then it is called an identity.

Root of an Equation – The value of unknown or variable for which the equation is true is known as the root of the equation. To find the roots of an equation means to solve the equations.

Degree of an Equation – The degree of an equation is the highest exponent of the variable x or variables (x, y, …) present in the equation is called the degree of an equation.

Linear Equation – An equation which involves power of an unknown quantity not higher than unity (one) is called a linear equation.

One variable Linear Equation – A linear equation in one variable (x, say) in which the highest degree of the variable x is 1. A linear equation in one variable is, in general, written as ax+by = c or ax = c. This equation is also called, "First degree equation in x" or simple equation.

Two variable equation – A linear equation in two variables (x, y, say) in which the highest degree of the variables x and y each is 1. A linear equation in two variables, is general, is written as ax+by+c = 0 or ax+by=d.

Three variable equation – A linear equation in three variables (x, y, z, say) in which the highest degree of the variables x, y and z each is 1. A linear equation in three variables, in general, is written as $a_1x+b_1y+c_1z=d$.

Types of Simultaneous Equations –

i) Linear Simultaneous Equations in two Variables – Two linear equations in two variables together are linear simultaneous equations in two variables, e.g.:

$$4x+y = 2$$
$$3x-5y = 18$$

ii) Linear Simultaneous Equations in three Variables – Three linear equations in three variables together are linear simultaneous equations in three variables, e.g.:

$$3x+5y-7z = 13$$
$$4x+y-12z = 6$$
$$2x+9y-3z = 20$$

iii) Specific type of Simultaneous Equations – The equations in other than linear form are called specific type equations, e.g.:
   i) quadratic equation : $ax^2 + bx + c = 0$
   ii) reciprocal equation : $\frac{a}{x} + \frac{b}{y} = c$

iii) $a\left(\frac{y}{x}\right) + c = by, etc.$

Characteristics of Simultaneous Equations –

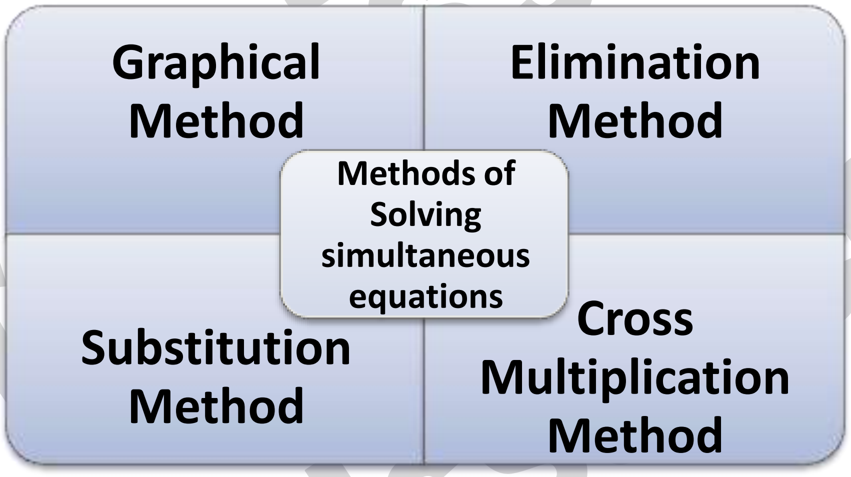
1) A system of linear equations in one variable is not taken under simultaneous equations.
2) The set of values of two variables x and y which satisfy each equation in the system of equations is called the solution of simultaneous equations.
   The solutions of two variable linear simultaneous equations may be –
   i) Infinitely many,
   ii) An unique solution, or
   iii) No solution.

3) For simultaneous equations –
   $a_1x + b_1y = c_1$ and $a_2x + b_2y = c_2$
   a. If $\frac{a1}{a2} = \frac{b1}{b2} = k and c1 = k$ C_2 then there are infinitely many solutions.
   b. If $\frac{a1}{a2} = \frac{b1}{b2} = c1 \neq kc_2$, then there is no solution.
   c. If $c_2 \neq 0$, then $c_1 = kc2 \rightarrow \frac{c1}{c2} = k, hence$

$$\frac{a1}{a2} = \frac{b1}{b2} = \frac{c1}{c2} \rightarrow infinitely many solutions$$

$$and\ \frac{a1}{a2} = \frac{b1}{b2} \neq \frac{c1}{c2} \rightarrow no solution$$

   d. If $\frac{a1}{a2} \neq \frac{b1}{b2}$, then there is an unique solution of the given system of equations.

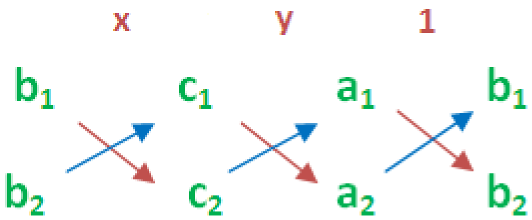


Solving by Elimination:
1) Write the equations in the same order. (line up the x's and y's)
2) Make the numbers in front of the $x's$ OR the $y's$ the same. (whichever seems easier)
3) Same signs: subtract one equation from the other. Different signs: add the equations together.
4) Solve the new equation to find x or y.
5) Substitute back into one of the original equations to find the other letter.

Solving by Substitution:
1) Rearrange one of the equations (if necessary) to make either $\boldsymbol{x}$ or $\boldsymbol{y}$ the subject.

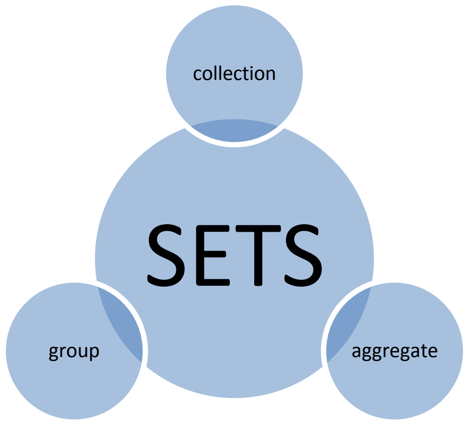
2) Substitute either x or y into the other equation.
3) Solve the new equation to find x or y.
4) Substitute back into your rearranged equation to find the value of the other letter.

Solving by Cross Multiplication :



SET THEORY

A **Set** is an unordered collection of objects, known as elements or members of the set.
An element 'a' belong to a set A can be written as '$a \in A$', '$a \notin A$' denotes that a is not an element of the set A.



If we have a set we say that some objects belong (or do not belong) to this set, are (or are not) in the set. We say also that sets consist of their elements.

Examples:
The set of students in this room; the English alphabet may be viewed as the set of letters of the English language; the set of natural numbers; etc

Representation Of a Set

Roster form
In this representation, elements are listed within the pair of brackets {} and are separated by commas

1. Let N is the set of natural numbers less than 5.
N = { 1 , 2 , 3, 4 }.
2. The set of all vowels in the English alphabet.
V = { a , e , i , o , u }.

Set builder form
In Set-builder set is described by a property that its member must satisfy.

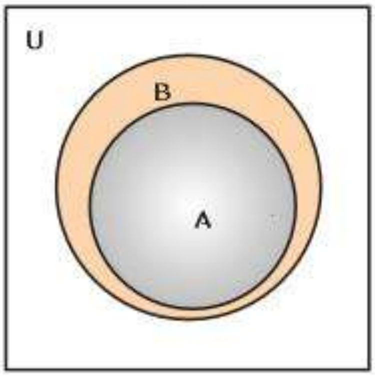
1. {x : x is even number divisible by 6 and less than 100}.
2. {x : x is natural number less than 10}.
3.V = {x: x is a vowel of English alphabet}
4. Let A be the set of natural numbers less than 7.
Then A = {x: x ∈N and 1<=x<=7}

Note:
Symbol ':' read as 'such that'

- **Finite** A set is said to be finite if its elements can be counted
- **Infinite**: it is said to be infinite if it is not possible to count upto its last element.
- **Empty set**: There is exactly one set, the empty set, or null set, which has no members at all.
- **Singleton set**: A set with only one member is called a singleton or a singleton set. (“Singleton of a”)
- **Disjoint** Two sets are said to be disjoint if they do not have any common element
- **Equal sets:** Two sets A and B are said to be equal if they contain the same elements. Every element of A is an element of B and every element of B is an element of A.

Subset

A set A is said to be **subset** of another set B if and only if every element of set A is also a part of other set B.
Denoted by ‘⊆‘.
‘A ⊆ B ‘ denotes A is a subset of B.
To prove A is the subset of B, we need to simply show that if x belongs to A then x also belongs to B.
To prove A is not a subset of B, we need to find out one element which is part of set A but not belong to set B.



A ⊆ B

‘U’ denotes the universal set.
Above Venn Diagram shows that A is a subset of B.

Universal set: A set which has all the elements in the universe of discourse is called a universal set.

Power Sets

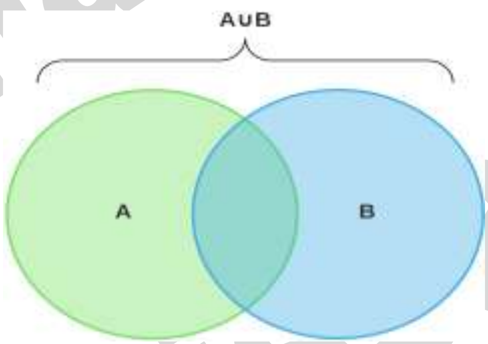
The power set is the set all possible subset of the set S. Denoted by P(S).
Example: What is the power set of {0,1,2}?
Solution: All possible subsets
{∅}, {0}, {1}, {2}, {0,1}, {0,2}, {1,2}, {0,1,2}.
Note: Empty set and set itself is also the member of this set of subsets..

Venn Diagram

British mathematician John Venn (1834–1883 AD) introduced the concept of diagrams to represent sets. According to him universal set is represented by the interior of a rectangle and other sets are represented by interior of circles.

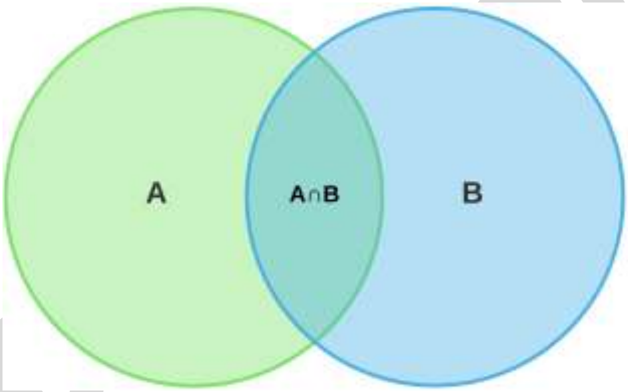
Union Of Sets

If A and B are only two sets then union of A and B is the set of those elements which belong to A or B. denoted by A U B



Intersection Of Sets

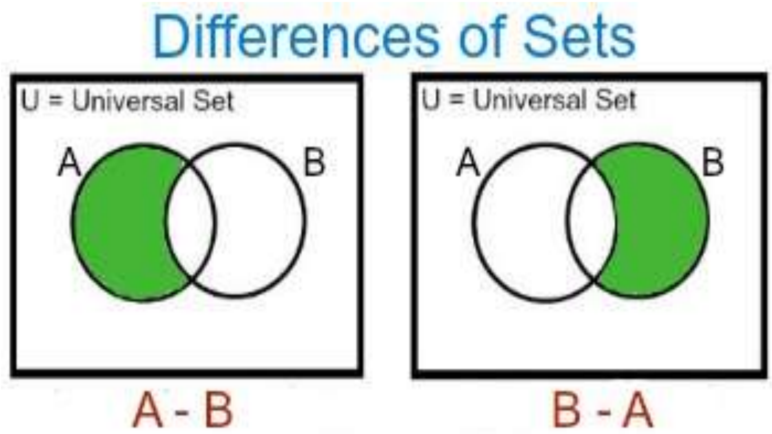
Consider the sets
A = {1, 2, 3, 4} and B = {2, 4, 6}
It is clear, that there are some elements which are common to both the sets A and B. Set of these Common elements is said to be intersection of A and B and is denoted by A∩B
Here
A∩B= {2, 4 }



Difference Of Sets

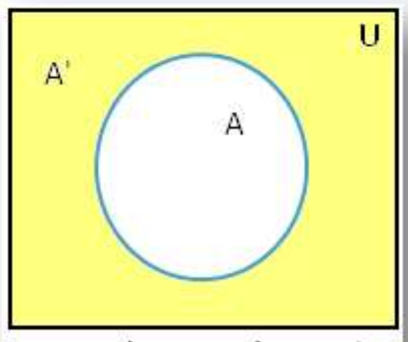
Consider the sets

A = {1, 2, 3, 4, 5} and B= {2, 4, 6}.
A new set having those elements which are in A but not B is said to be the difference of sets A and B and it is denoted by A−B.
∴A−B= {1, 3, 5}



Compliment or Negation of a set :

Consider the set U={1,2,3,4,5} and set A={2,4}, so A^c (not A) = {1,3,5}, i.e. all elements of U which are not in A.



Complement of A or A'

Definition of De Morgan's law:

The complement of the union of two sets is equal to the intersection of their complements and the complement of the intersection of two sets is equal to the union of their complements. These are called **De Morgan's laws**.

For any two finite sets A and B;

(i) (A U B)' = A' ∩ B' (which is a De Morgan's law of union).

(ii) (A∩ B)' = A' U B' (which is a De Morgan's law of intersection).

UNIT-V
QUADRATIC EQUATIONS

A quadratic equation is usually defined as $ax^2 + bx + c = 0$, where a, b, and care the real terms and a is not equal to 0. Because if a = 0, then the equation will be linear.

$$ax^2 + bx + c$$

$\underbrace{ax^2 + bx}_{\text{coefficients}} + \underbrace{c}_{\text{constant}} = 0$ $\underbrace{2x^2 + 3x}_{\text{coefficients}} + \underbrace{4}_{\text{constant}} = 0$

a and **b** are coefficients and **c** is a constant. The one factor that identifies these expressions as **quadratic** is the exponent 2. The first term must always be **ax²**, and **a** cannot be 0.

Equation	Is it Quadratic?	Explanation
$3x^3 - 4x + 5$	No	The first term is raised to the 3rd power. It must be raised to the 2nd power in order to be quadratic.
$5x^2 - 4x + 2$	Yes	This equation is in the correct form: $ax^2 + bx + c$
$7x^2 = 49$	Yes	This equation can be rewritten as: $7x^2 - 49$. In this equation, *b* is 0. *B* or *c* can be 0; however, *a* cannot be 0.
$2x^2 = 8x - 3$	Yes	This equation can be rewritten as $2x^2 - 8x + 3$ which would then be in the correct form of: $ax^2 + bx + c$.

Roots of a Quadratic Equation: A quadraticeequationwillalwayshavetwovalues/solutionsorrootsofx.

Definition of discriminant: For the quadratic equation $ax^2 + bx + c = 0$, $a \neq 0$; the expression $b^2 - 4ac$ is called discriminant and is, in general, denoted by the letter 'D'.
Thus, discriminant $D = b^2 - 4ac$

Note:

Discriminant	Nature of roots
$b^2 - 4ac = 0$	Real and equal
$b^2 - 4ac > 0$	Real and unequal
$b^2 - 4ac < 0$	Not real

There are 3 methods of solving a quadratic equation, namely :

1. Factorization Method
2. Completing the square method
3. Sridhacharya's method

Factorization

The first and simplest method of solving quadratic equations is the factorization method. Certain quadratic equations can be factorised. These factors, if done correctly will give two linear equations in x. Hence, from these equations, we get the value of x. Let's see an example and we will get to know more about it.

Examples of Factorization

Example 1: Solve the equation: x2 + 3x – 4 = 0
Solution: This method is also known as splitting the middle term method. Here, a = 1, b = 3, c = -4. Let us multiply a and c = 1 * (-4) = -4. Next, the middle term is split into two terms. We do it such that the product of the new coefficients equals the product of a and c.

We have to get 3 here. Consider (+4) and (-1) as the factors, whose multiplication is -4 and sum is 3. Hence, we write x2 + 3x – 4 = 0 as x2 + 4x – x – 4 = 0. Thus, we can factorise the terms as: (x+4)(x-1) = 0. For any two quantities a and b, if a×b = 0, we must have either a = 0, b = 0 or a = b = 0.

Thus we have either (x+4) = 0 or (x-1) = 0 or both are = 0. This gives x+4 = 0 or x-1 = 0. Solving these equations for x gives: x=-4 or x=1. This method is convenient but is not applicable to every equation. In those cases, we can use the other methods as discussed below.

Completing the Square Method

Each quadratic equation has a square term. If we could get two square terms on two sides of the quality sign, we will again get a linear equation. Let us see an example first.

Example 2: Let us consider the equation, 2x2=12x+54, the following table illustrates how to solve a quadratic equation, step by step by completing the square.
Solution: Let us write the equation 2x2=12x+54. In the standard form, we can write it as: 2x2 – 12x – 54 = 0. Next let us get all the terms with x2 or x in them to one side of the equation: 2x2 – 12 = 54

In the next step, we have to make sure that the coefficient of x2 is 1. So dividing throughout by the coefficient of x2, we have: 2x2/2 – 12x/2 = 54/2 or x2 – 6x = 27. Next, we make the left hand side a complete square by adding (6/2)2 = 9 i.e. (b/2)2where 'b' is the new coefficient of 'x', to both sides as: x2 – 6x + 9 = 27 + 9 or x2 – 2×3×x + 32 = 36. Now we can write it as a binomial square:

- (x-3)2 = 36; Take square root of both sides
- x – 3 = ±6; Which gives us these equations:
- x = (3+6) or x = (3-6) or x = 9 or x = -3

This is known as the method of completing the squares.

Quadratic Equation Formula

There are equations that can't be reduced using the above two methods. For such equations, a more powerful method is required. A method that will work for every quadratic equation. This is the general quadratic equation formula. We define it as follows: If ax2 + bx + c = 0 is a quadratic equation, then the value of x is given by the following formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Just plug in the values of a, b and c, and do the calculations. The quantity in the square root is called the discriminant or D. The below image illustrates the best use of a quadratic equation.

Example 3: Solve: x2 + 2x + 1 = 0

Solution: Given that a=1, b=2, c=1, and
Discriminant = b2 – 4ac = 22 – 4×1×1 = 0

Using the quadratic formula, $x = (-2 \pm \sqrt{0})/2 = -2/2$
Therefore, $x = -1$

LINEAR PROGRAMMING

Origin and Development
This technique is first originated and invented by Russian Matehmatician L.B. Contarowitch. In 1947 George Dantzig and his associates found out this technique for solving military planning problems while they are working on project for US Air Force.

Meaning and Definition –
Linear programming consists of two words 'linear' and programming. The wore 'linear' establishes certain relationship among different variables whereas the word 'programming' indicates a way to get the desired results by the optimum use of a variable resources.

In reality linear programming is a mathematical technique in which selecting the best possible alternative among a number of alternatives available with the management. The chosen alternative is said to be the best because is involves maximization of profits or minimization of cost, i.e., maximum profit at minimum cost.

Definition –

Following are the definitions of Linear Programming –
According to William Fox, "Linear programming is a planning technique that permits some objective functions to be minimized or maximized within the frame work of given situational restrictions."
According to Bowman and Fetter, "It is a method of planning whereby some objective functions is minimized or maximized while at the same time satisfying the various restrictions placed on the potential solution."

Features of Linear programming –
1) ***Linear relationship*** – Linear programming can be used when there is a certain relationship among different variables.
2) ***Objectives and Goal*** – In Linear programming, there must be a well-defined objective through which profit is maximized and cost is minimized.
3) ***Presence of alternatives*** – In linear programming there must be alternative course of action for solving the problem of decision.
4) ***Limited Resources*** – Linear programming is used only, when the resources, i.e., labour capital etc. must be in limited supply.
5) ***Problems expressed quantitatively*** – In linear programming the problems are capable of being expressed quantitatively.

Example: Consider a chocolate manufacturing company which produces only two types of chocolate – A and B. Both the chocolates require Milk and Choco only. To manufacture each unit of A and B, following quantities are required:

- Each unit of A requires 1 unit of Milk and 3 units of Choco
- Each unit of B requires 1 unit of Milk and 2 units of Choco

The company kitchen has a total of 5 units of Milk and 12 units of Choco. On each sale, the company makes a profit of

- Rs 6 per unit A sold
- Rs 5 per unit B sold.

Now, the company wishes to maximize its profit. How many units of A and B should it produce respectively?

Solution: The first thing I'm gonna do is represent the problem in a tabular form for better understanding.

	Milk	Choco	Profit per unit
A	1	3	Rs 6
B	1	2	Rs 5
Total	5	12	

Let the total number of units produced of A be = X

Let the total number of units produced of B be = Y

Now, the total profit is represented by Z

The total profit the company makes is given by the total number of units of A and B produced multiplied by its per unit profit Rs 6 and Rs 5 respectively.

Profit: Max Z = 6X+5Y

which means we have to maximize Z.

The company will try to produce as many units of A and B to maximize the profit. But the resources Milk and Choco are available in limited amount.

As per the above table, each unit of A and B requires 1 unit of Milk. The total amount of Milk available is 5 units. To represent this mathematically,

$$X+Y \leq 5$$

Also, each unit of A and B requires 3 units & 2 units of Choco respectively. The total amount of Choco available is 12 units. To represent this mathematically,

$$3X+2Y \leq 12$$

Also, the values for units of A can only be integers.

So we have two more constraints, $X \geq 0$ & $Y \geq 0$

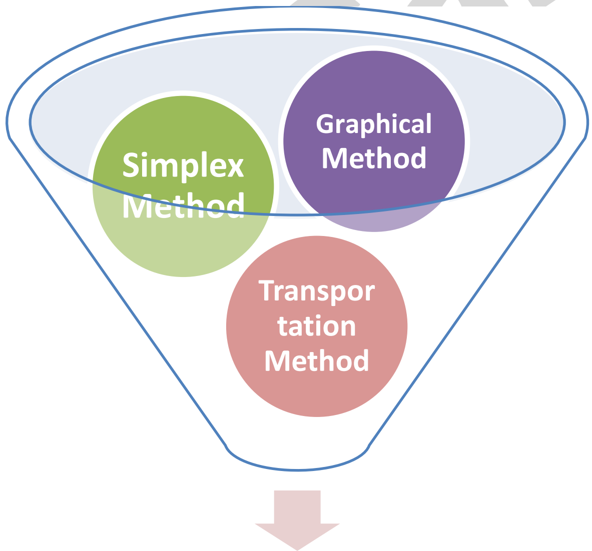
For the company to make maximum profit, the above inequalities have to be satisfied.

This is called formulating a real-world problem into a linear programming mathematical model.

Common terminologies used in Linear Programming

Let us define some terminologies used in Linear Programming using the above example.

- **Decision Variables:** The decision variables are the variables which will decide my output. They represent my ultimate solution. To solve any problem, we first need to identify the decision variables. For the above example, the total number of units for A and B denoted by X & Y respectively are my decision variables.
- **Objective Function:** It is defined as the objective of making decisions. In the above example, the company wishes to increase the total profit represented by Z. So, profit is my objective function.
- **Constraints:** The constraints are the restrictions or limitations on the decision variables. They usually limit the value of the decision variables. In the above example, the limit on the availability of resources Milk and Choco are my constraints.
- **Non-negativity restriction:** For all linear programs, the decision variables should always take non-negative values. Which means the values for decision variables should be greater than or equal to 0.



Methods of Solving Linear Programming Problem

1) **Graphical Method –**
   Graphic method is used for solving the problems having two or three variables. In practice two variable cases are easy to solve by this method because three dimensional geometry becomes too complicated to find accurate results.
2) **Simplex Method –**
   Simplex method of linear programming is a very important technique to solve the various linear problems. Under this method, algebraic procedure is sued to solve any problem.

Several variables (more than two) can be used under this method. Simplex method is more complex method. In this method various linear problems can be solve with the help of computer.

3) **Transportation Method –**
Transportation method is used to know the minimum cost of transportation of a product from various origin to different distribution and consumption centers. The quantity to the supplied from various origins or production centers, the quantity demanded at various destinations and the cost of transportation per unit from a particular origin to some specific destination are assumed to be known and with the help of these information the optimal schedule is prepared.